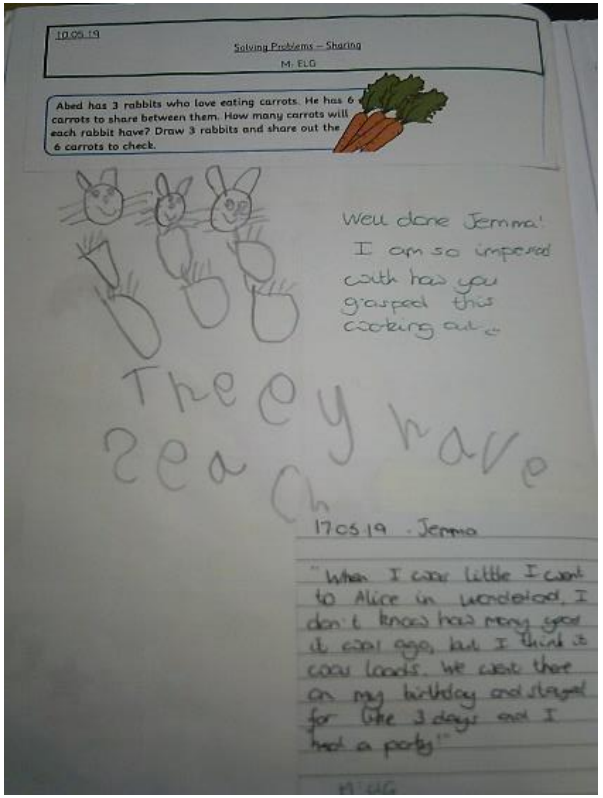
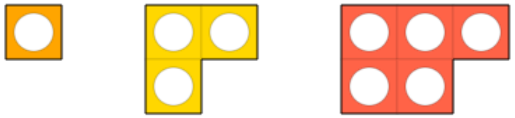
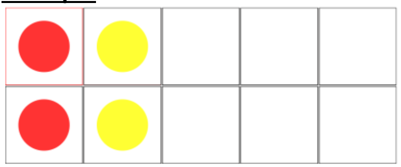
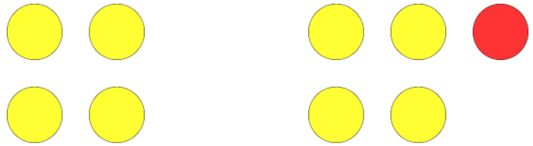
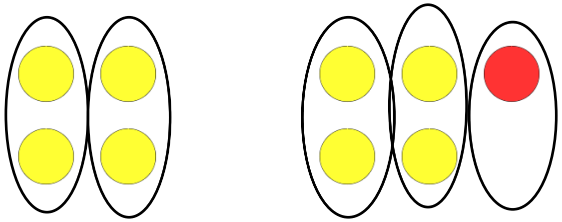
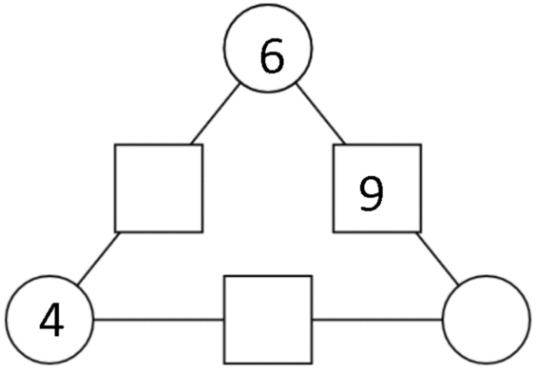
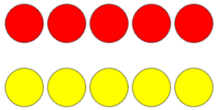
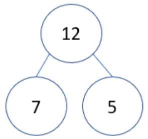
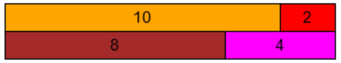
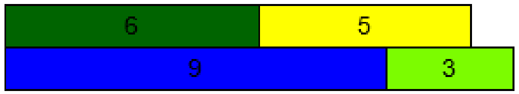
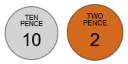
# Progression in Reasoning

| EYFS | Key learning | Examples of reasoning | Vocabulary and thinking terms |
|---|---|---|---|
| | To engage with rich contexts for exploring mathematical ideas, making useful connections and developing mathematical skills and concepts<br><br>Make connections to the theme and connect the learning to the play.<br><br> <br><br>- Matching quantity to numerals<br>- Sequencing and ordering<br>- Concept of one more<br>- Numbers bonds to 5 | Stories – planning your way through a story, making choices.<br>Thematic approaches (seasons) for sorting and matching to criteria.<br><br>Environment connections (home and role play, small world, construction and malleable play)<br><br>- Today we went on a dinosaur hunt. Look at what we found. T-Rex tracks! What can you spot on the T-Rex tracks? How could you sort these? Why did you choose to do this? What if we added one more foot print? What could it be?<br><br> | Tell me<br>Why did you choose?<br>How many do you have altogether?<br>What if we had more?<br>What is we had something different?<br>What if you had one other the same?<br>Is there another way of doing it?<br>What would you like to do next? |

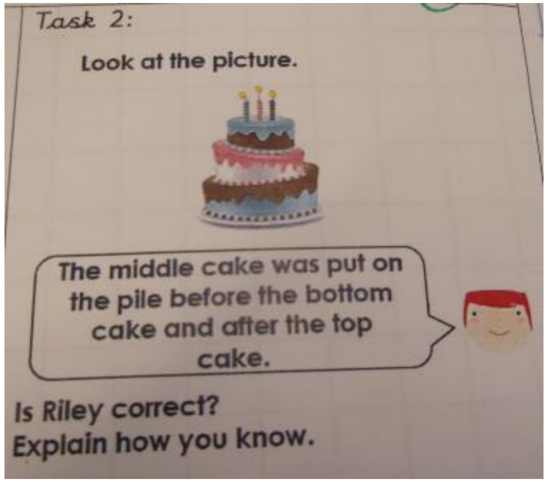
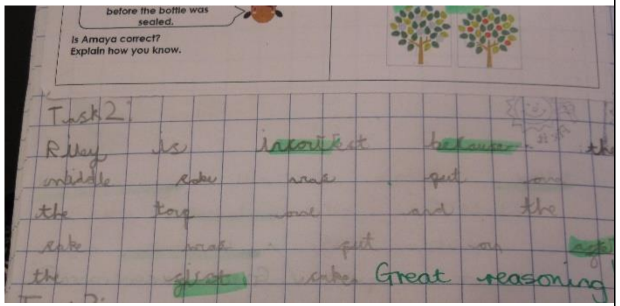
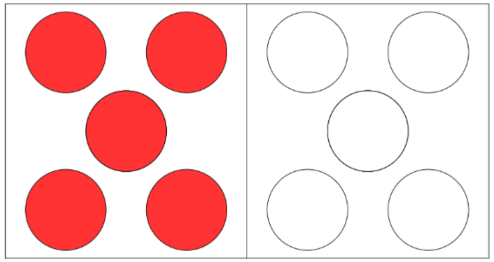
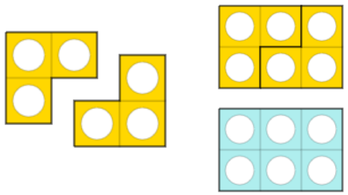
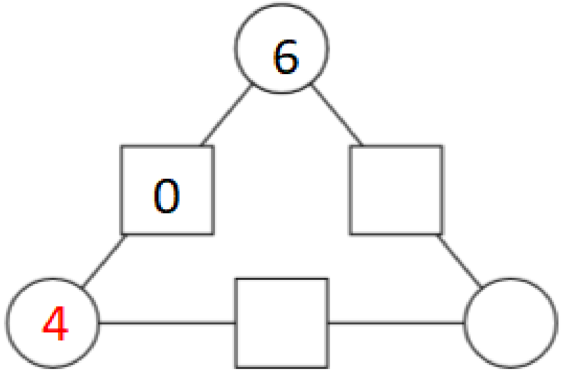
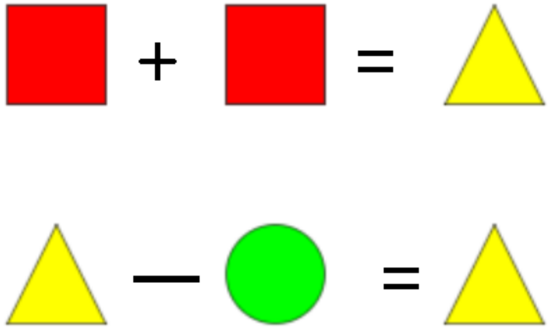
# Progression in Reasoning

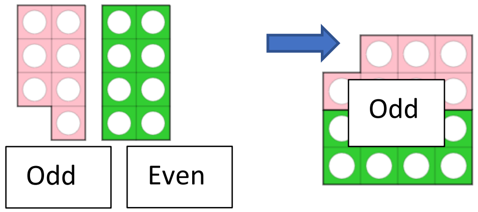
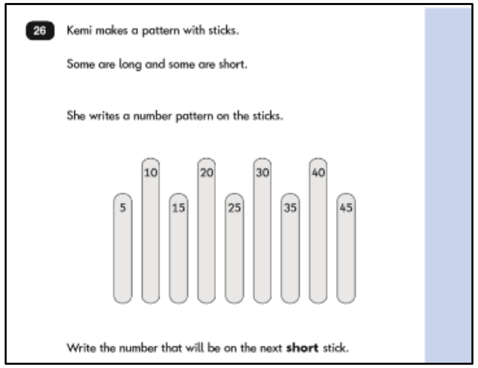
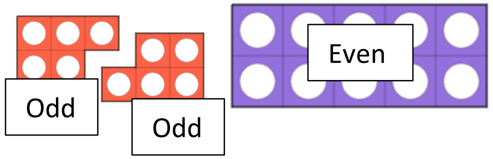
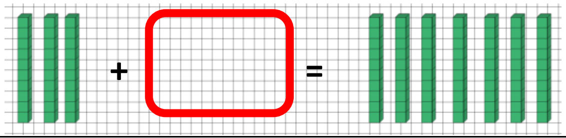
| | **Models of Proof/Evidence** | **Specialising**<br>is about starting with something general and seeing what it tells us about a specific case. | **Generalising**<br>is about starting with specific cases and becoming less specific. |
|---|---|---|---|
| | The children had to share the 6 carrots between 3 rabbits and they stated how many each rabbit should have.<br> | **ODDS**<br><br>1, 3, 5, 7, 9 consecutive odds<br>9 is an odd number because the unit digit is odd<br>13 has two odd digits<br><br>**Complete the next 3 numbers in the sequence. Jean says I am going to put a 12 next. Explain why this is incorrect.**<br><br>**EVENS**<br><br>2, 4, 6, 8 consecutive evens<br>16 is an even number because the unit digit is even<br><br>**Example**<br><br>**2 4 _____ _____ _____**<br>**Complete the sequence**<br><br>**Prove how this model shows that 7 cannot be an even number.** | **Odd - A number or quantity that cannot be divided equally into two groups.**<br><br>**Even - A number or quantity that can be divided equally into two groups.**<br><br>**The model proves a general rule. Is it odd or even?**<br><br>**Annotate the diagram to prove this generalisation.**<br><br>**What if I added 4 more counters to the diagrams, what would change and what would stay the same?** |

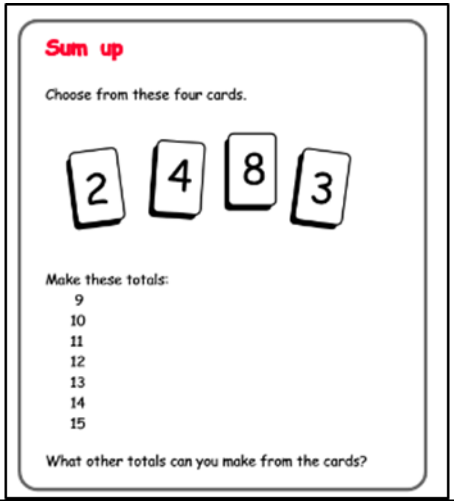
# Progression in Reasoning

| Year 1 | Key learning | Examples of reasoning | Vocabulary and thinking terms |
|---|---|---|---|
| | To apply conceptual knowledge to recognise patterns and relationships, **to show** results using clear mathematical models such as practical apparatus, diagrams or number sentences.<br><br>Fill in the missing boxes so the sum of the numbers of each line totals 20.<br><br>What can you tell me about the numbers in the squares? | Subitising and representing amounts.<br><br>**Make 12 using concrete resources. Show as many ways you can to make different amounts.**<br><br>5 + 5 + 2 =12<br><br>**Represent 12 using pictorial representations and diagrams.**<br><br>**Equality and inequality.**<br><br>6 + 5 does not equal 9 + 3<br><br>Use role play contexts to set up scenarios for children to apply knowledge and connect mathematical ideas.<br><br>How many ways can you make 12p? | Show me different ways to...<br>True or false<br>What is the same and what is different?<br>Spot the mistake<br>What comes next?<br>What do you notice?<br>Convince me<br>Why and why not...<br>Find one ... Find all...<br><br>Are these amounts equal? |

# Progression in Reasoning

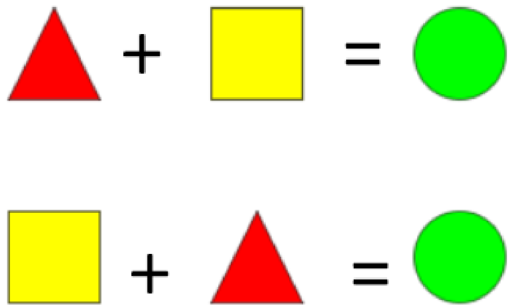
| | **Models of Proof/Evidence** | **Specialising**<br>is about starting with something general and seeing what it tells us about a specific case. | **Generalising**<br>is about starting with specific cases and becoming less specific. |
|---|---|---|---|
| | Mathematical explanation – why is something true or not true?<br><br> | Adding or subtracting zero leaves a number unchanged<br>9 + 0 = 9<br>9 - 0 = 9<br><br>13 + 0 = 13<br>13 – 0 = 13<br><br>Add and subtract 0 to this number.<br>*Simon says when you add zero you must colour in the next circle. Is he correct? Explain your reasoning.*<br>Odd + odd = even<br> | 12 + 0 = 12<br>12 – 0 = 12<br>The rule for adding or subtracting 0 from any number is that the number will remain unchanged.<br>Explain how you now that the number the bottom circle has to be a 4 if all sides total 10.<br><br>Explain how do you know that the green circle has a value of 0.<br> |

| | | | |
|---|---|---|---|
| | | 3 + 3 = 6, so odd + odd = even | Every time I add an odd number to an even number, I make an odd total. Is this true?<br><br> |
| **Year 2** | **Key learning** | **Examples of reasoning** | **Vocabulary and thinking terms** |
| | To apply conceptual knowledge to recognise patterns and relationships, **to explain** results using clear mathematical models such as practical apparatus, diagrams or number sentences.<br><br>26 Kemi makes a pattern with sticks.<br>Some are long and some are short.<br>She writes a number pattern on the sticks.<br><br>Write the number that will be on the next **short** stick. | Mathematical modelling – **effect of 0, odds and evens** (generalisations).<br>Children predict and make a summary of their findings.<br>'odd + odd = odd' Modelling the counter example<br><br><br>**Empty box equations**<br>10+2 = 9+___ convince me that the number in the missing box is 3.<br><br><br>SATS focus.<br><br> | Show me<br>Convince me<br>Model of proof<br>Satisfying a rule<br>Why and why not<br>What else do you know? Use a fact to prove or disprove<br>True or false<br>What number is missing?<br>Odd one out<br>Find one … Find all…<br>Undoing (working backwards) using the inverse |

<table>
<tr>
<td></td>
<td>Is it possible to make the odd totals without using the 3?<br><br>
Sum up<br>
Choose from these four cards.<br>
2 4 8 3<br>
Make these totals:<br>
9<br>
10<br>
11<br>
12<br>
13<br>
14<br>
15<br>
What other totals can you make from the cards?</td>
<td>Develop strategies for undoing<br><br>
? → x2 → +6 → 24<br><br>
24<br><br>
Odd one out – 3 different numbers<br>
40, 65, 71 - which is the odd one out?<br>
Explain your reasoning (justify and prove).<br><br>
71 – it's not a multiple of 5<br>
40 – it's a multiple of 10, it's even<br>
65 – it has one odd and one even digit</td>
<td></td>
</tr>
<tr>
<td></td>
<td><u>Models of Proof/Evidence</u></td>
<td><u>Specialising</u><br>is about starting with something general and seeing what it tells us about a specific case.</td>
<td><u>Generalising</u><br>is about starting with specific cases and becoming less specific.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>35 + __ = 75<br>
Use the inverse to solve this calculation.<br>
<u>Example</u><br>
<table><tr><td colspan="2">75</td></tr><tr><td>35</td><td>?</td></tr></table><br>
Sally says she thinks the missing number is 40.<br>
Simon thinks the missing number is 4.<br>
Who is correct? Explain your answer using mathematical vocabulary of inverse.</td>
<td><u>Addition and Subtraction</u><br>
The inverse of addition is subtraction.<br>
The inverse of subtraction is addition.</td>
</tr>
</table>

# Progression in Reasoning

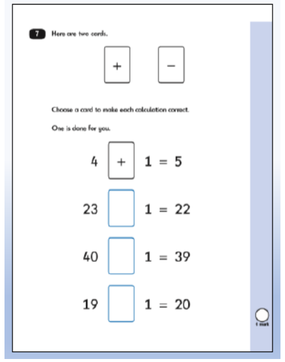
Use this model of proof to show how any addition can be reordered.




**Extension**

**Simon was incorrect. Can you explain to Simon why he is incorrect?**

| |
|---|
| Addition makes a larger total.<br><br>Subtraction makes the answer smaller |


7 Here are two cards.

+ −

Choose a card to make each calculation correct.

One is done for you.

4 + 1 = 5

23 ☐ 1 = 22

40 ☐ 1 = 39

19 ☐ 1 = 20

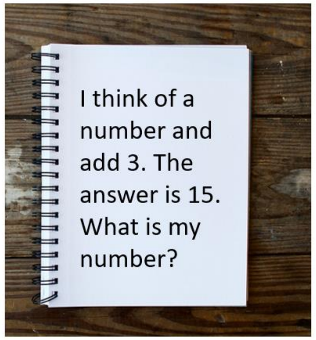
**Rules of Divisibility**

**Any even number will divide equally by 2**

**20 ÷ 2 = 10**
**10 x 2 = 20**

**All multiples of 5 end in a 5 or 0**
**35 ÷ 5 = 7**
**5 x 8 = 40**

**All multiples of 10 end in a 0**




? → +3 → 15

12 ← -3 ← 15

**Multiplication and division**

**The inverse of multiplication is division**

**The inverse of division is multiplication**

**Example**

Why and why not?

? → +6 → ÷ 2 → 9

12 ← - 6 ← x 2 ← 9

5 ← - 6 ← + 2 ← 9

The missing value cannot be 5 because the inverse of division is multiplication.

**Rules of Divisibility**

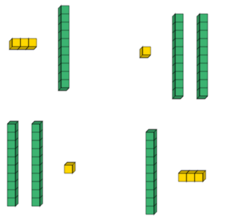
2 x 3 = 6, 10 x 2 = 20

**Any even number will divide equally by 2; even**

5 x 5 = 25, 5 x 6 = 30

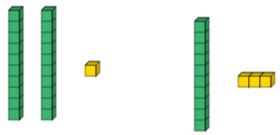
**100 ÷ 10 = 10**
**10 x 10 =100**

**Commutative law**



13 + 21 is the same as 21 + 13.

**Non-commutative rule**



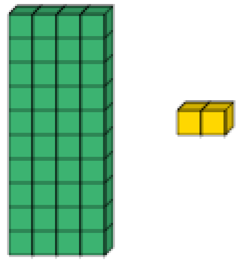
21 – 13 cannot be reordered to 13 – 21.

**All multiples of 5 end in a 5 or 0; odd or even**

10 x 7 = 70, 8 x 10 = 80
**All multiples of 10 end in a 0; even**

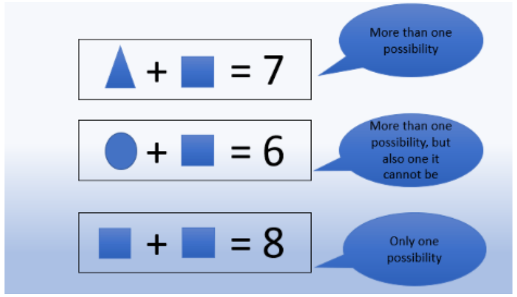
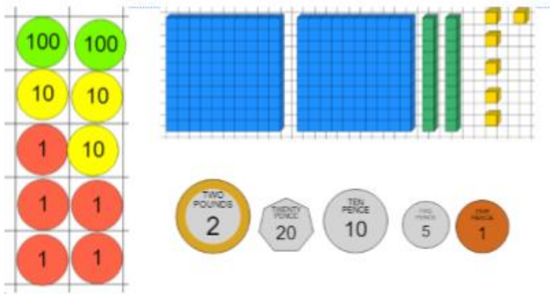
**Commutative law**
**Additions can be done in any order; the answer will be the same**

**Non-commutative rule**
**Subtractions cannot be reordered**



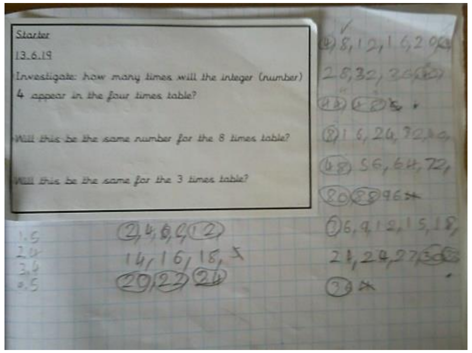
I cannot subtract 56 from 42 because I would need to take 5 tens and 6 ones.

# Progression in Reasoning

| Year 3 | Key learning | Examples of reasoning | Vocabulary and thinking terms |
|---|---|---|---|
| Y3 | To apply conceptual knowledge to use patterns, relationships and properties of number **to begin with generalising**. To explain results using clear mathematical models such as practical apparatus, diagrams or number sentences **as models of proof**. Finding starting points with reasoned argument for logic.<br><br> | **Empty box questions** with unknowns and different patterns of variables.<br><br>24+ ☐= 15+15 – unknown<br><br>24+☐ = 15+☐ – patterns of variables<br>0 and 9<br>1 and 10<br>2 and 11<br><br><u>Odd one out</u><br><br><br>Explain your choice. | Show me<br>Convince me<br>Why and why not, what if<br>Model of proof<br>Satisfying a rule<br>What else do you know? Use a fact<br>True or false<br>What number is missing?<br>Odd one out<br>Undoing (working backwards)<br>Always, sometimes, never.<br>Variables (find all/enough)<br>Logical reasoning |

# Progression in Reasoning

## Models of Proof/Evidence




## Specialising

is about starting with something general and seeing what it tells us about a specific case.

**Multiples**

4, 6, 8, 10 are multiples of 2
16, 24, 32 are multiples of 2, 4 and 8
Multiples of 4 and 8 are even.
Multiples of 3 are odd or even.
3, 6, 9, 12…

**How many times does the digit 4 appear between 0-100 when counting in multiples of 4?**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Extension**

Sam says that 4 doubled is 8. So the number of times the digit 8 appears between 0-100 when counting in groups of 8 will be doubled. Can you prove Sam's theory? Use jottings to explain your thinking.

## Generalising

is about starting with specific cases and becoming less specific.

**Multiples**

A multiple is the result of multiplying a number by a whole number several times. They are in the same family of multiples.
Even tables have even multiples.
Odd tables have odd and even multiples.

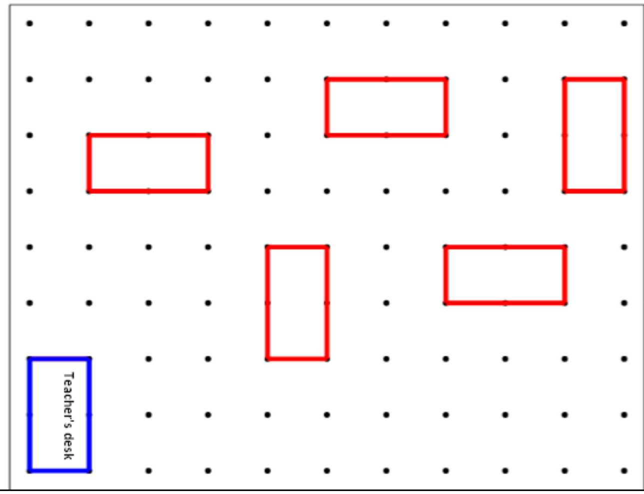
**Multiples of 3**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

**Multiples of 4**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Here is Holly's new classroom.



| | | | |
|---|---|---|---|
| | | **<u>Rules of divisibility</u>**<br><br>87 ÷ 3 = 29<br><br>8+7 = 15 so this must divide by 3 because 15 is a multiple of 3. | Each table can sit 6 children. Holly says to work out how many pencils the whole class has is very simple. All you need to do is to place one pencil on each table until every table has 6 pencils.<br><br>Question - Explain why this is an inefficient method.<br><br>**Rules of divisibility**<br><br>A number will divide by 3 if the digit sum is a multiple of 3.<br><br><br>**Sam and Tim have the same amount in their wallet. Sam says “I can’t share mine between him and his friends.” But Tim disagrees and says he can share it evenly. Explain who is correct.** |

**Unknowns and variables**

**Unknowns**

**An unknown is a set value to fit a statement.**

Find the value of the Y

Y — X 4 — 32

Find the odd one out and explain your answer.

Y — X 4 — 32

Y — X 3 — 24

Y — X 8 — 48

Extension

Sam changes the value of Y. Y = 5

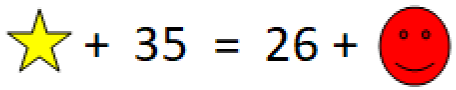
How does this change the outcome?

**Variables**

**A variable refers to a set of changing values.**

**Example**

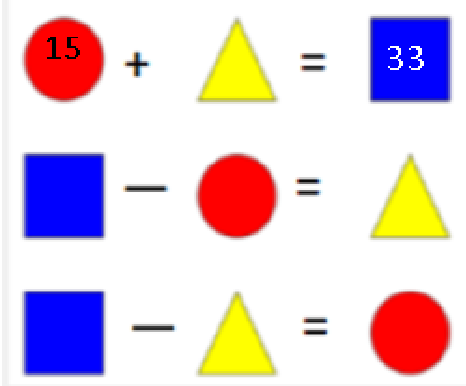
Find 3 different solutions to this problem.




Simon says he has to have an answer that is odd. Find 3 different solutions to this problem.

**Unknown**

**An unknown is a set value to fit a statement.**

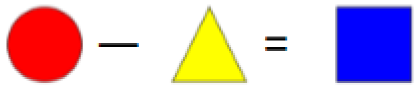



**Explain how using the inverse will help you find out the value of these digits. Remember to use examples to support your answer.**

Extension

True or False.

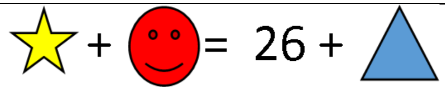
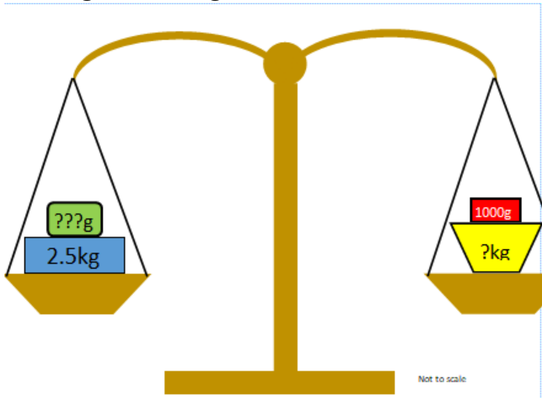
Sally says 'Using the inverse operation will allow you to find all the missing values.' Use this diagram to support you.



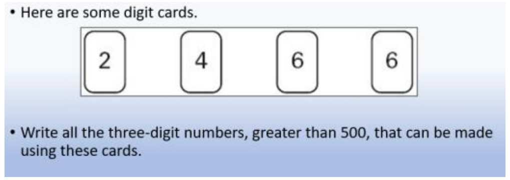
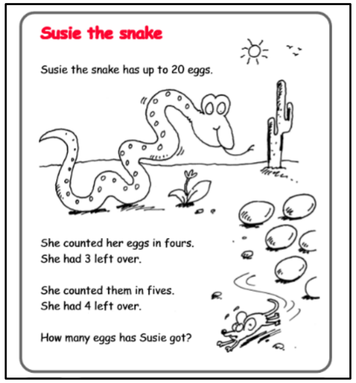
Explain your answer using mathematical vocabulary: Inverse operation, greater than, less than,

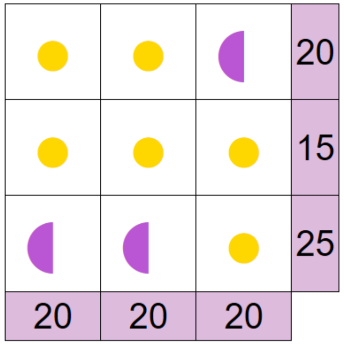
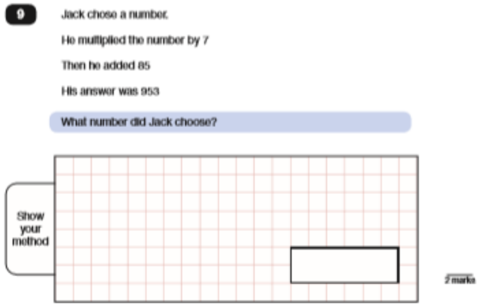
**Variable**

**A variable refers to a set of changing values.**

| | | | |
|---|---|---|---|
| | | <br><br>**Extension – Always, Sometimes, Never**<br><br>Holly says ‘to get an answer greater than 50, the star and face would have to bigger than 25. Find enough examples to satisfy your judgements. | **Impossible! Find the RIGHT answer. Find the WRONG answer and find the IMPOSSIBLE answer.**<br><br>Sally says “The values of the green and yellow parcels will always be 1 and half kg because 2.5kg – 1kg = 1.5kg”<br><br>Simon says “The values will always change depending on the total weight of each side of the scale.”<br><br>Sam says “To find the missing values you have to add the missing values together.”<br><br> |

# Progression in Reasoning

| Year 4 | Key learning | Examples of reasoning | Vocabulary and thinking terms |
|---|---|---|---|
| | To apply conceptual knowledge to use patterns, relationships and properties of number to **draw conclusions and make general statements**. **Lines of enquiry are generated and justified with mathematical models**. To explain results clearly using appropriate **representations and communications to offer a proof**.<br><br><br><br><br>Jenny is thinking of a number. She says,<br><br>“My number is a multiple of 4. It is also 3 less than a multiple of 5”<br><br>Find three different numbers that fit Jenny’s description.<br><br><br> | **Unknowns and variables linked to multiplication at this stage in Year 4.**<br><br>**Why or why not?**<br>Doubling a number is always bigger<br>Any number that is divisible by 3 is also divisible by 6.<br><br>Pupils need to know that they only need one example when disproving a statement that can never be true.<br>**You cannot change the order of a times table.**<br><br>**Correspondence problems**<br>How many different possibilities?<br>‘5 t-shirts and 3 trousers make 15 possibilities.”<br><br>**Lines of enquiry**<br>‘If I add 2 or more consecutive numbers, I can make all the counting numbers from 3 to 20’. Children need to know that lines of enquiry are not just one answer, they need to be able generalise and prove.<br><br>**Rules of Divisibility**<br>Divisible by 4, if the last two numbers are divisible by 4 the whole number is. 7**44**<br><br>**Associative law**<br><br>**When adding it doesn't matter how we group the numbers (i.e. which we calculate first).** | Show me<br>Convince me<br>Model of proof<br>Satisfying a rule<br>What else do you know? Use a fact<br>True or false<br>What number is missing?<br>Odd one out<br>Undoing (working backwards)<br>Always, sometimes, never.<br>Variables (find all/enough)<br>Logical reasoning<br>Undoing<br>Why?<br>Why not?<br>Always, sometimes, never<br>Make and create general statements. |

| | | | |
|---|---|---|---|
| | <br><br>The value of the yellow circle = 5. | **Example addition: (6 + 3) + 4 = 6 + (3 + 4)**<br>**Because 9 + 4 = 6 + 7 = 13**<br><br>**Also when multiplying it doesn't matter how we group the numbers.**<br><br>**Example multiplication: (2 × 4) × 3 = 2 × (4 × 3)**<br>**Because 8 × 3 = 2 × 12 = 24** | |
| | **<u>Models of Proof/Evidence</u>** | **<u>Specialising</u>**<br>is about starting with something general and seeing what it tells us about a specific case. | **<u>Generalising</u>**<br>is about starting with specific cases and becoming less specific. |
| | <br>9 Jack chose a number.<br>He multiplied the number by 7<br>Then he added 85<br>His answer was 953<br>What number did Jack choose?<br>Show your method<br>2 marks<br><br>**<u>Models of Proof</u>** | **<u>Specialising</u>**<br><br>1, 2, 3, 4, 6, 9, 12, 18 and 36 are all factors of 36.<br><br>-1, -2, -3, …..-15 | **<u>Generalising</u>**<br><br>*A factor is a number that will divide equally into a larger number.*<br><br>*A negative number is a number that is less than zero. Negative numbers are opposite to positive numbers.* |

# Progression in Reasoning

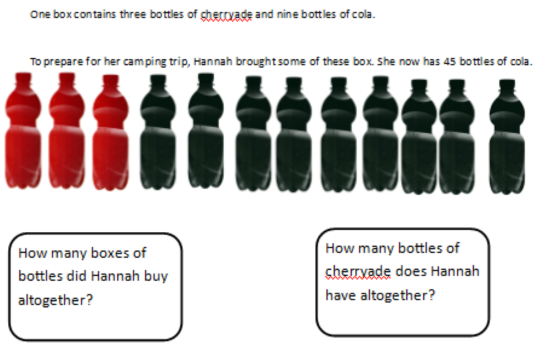
| | | | |
|---|---|---|---|
| ▶ | ▼ | ▼ | 16.7 |
| ▶ | ▼ | ▶ | 10.6 |
| ▶ | ▼ | ▼ | 16.7 |
| 4.5 | 22.8 | 16.7 | |

**Correspondence Problems**

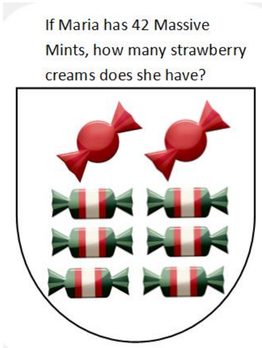

**42 ÷ 6 = 7**

**So 2 x 7 = 14 strawberry creams**

**What if there were 84 Massive Mints in a bag, how many Strawberry creams would there be?**




**Correspondence Problems**

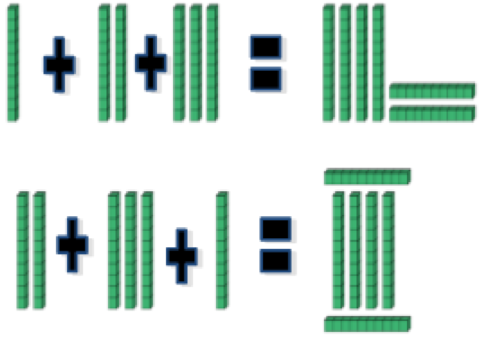



**Explain the rule for solving this correspondence problem.**

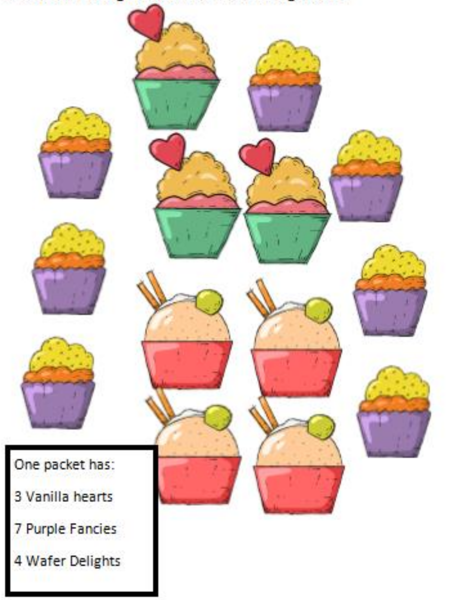
**Using this rule, write a set of instructions for a year 3 child, explaining how to solve this problem.**

**Associative law**

**Example addition: (10+20) + 30 = 20+30+10 = __ + ___ + ___**
**Because 10+20 = 30 + 30 = 60**

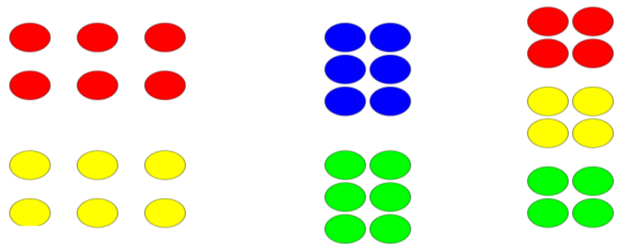


If Any has 36 Vanilla hearts, how many Purple Fancies and Wafer delights does he have altogether?

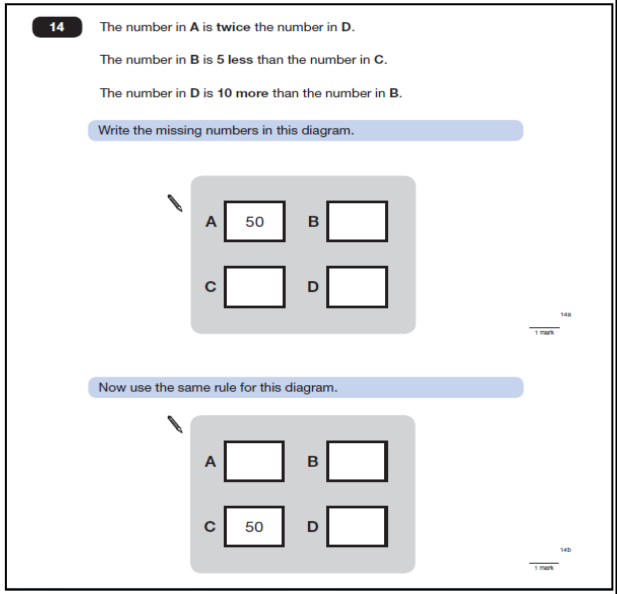
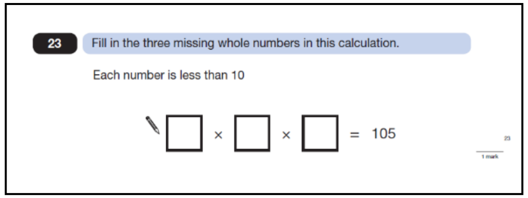



**Associative Law**
**When adding it doesn't matter how we group the numbers (i.e. which we calculate first).**

**Explain how these diagrams prove that when multiplying it doesn't matter how we group the numbers.**



# Progression in Reasoning

| Year 5 | Key learning | Examples of reasoning | Vocabulary and thinking terms |
|---|---|---|---|
| Y5 | To apply conceptual knowledge to make generalisations, **conjecture relationships and provide sophisticated models of proof**, including enquiry and **reasoned argument**.<br><br><br><br> | Generalise and develop a conjecture<br>**Tests of divisibility**<br>Ensure that all children know the tests of divisibility (will it need a remainder?)<br>2 – even<br>5 – 5,0<br>10 - 0<br>436 ÷ 3 =<br>To know that the digit sum must add to a multiple of 3 to divide by 3 without a remainder.<br><br>**6 –**if a number is even and adds to a multiple of 3 then the number will divide by 6.<br>**9 –** if the number adds to a multiple of 9 it will divide by 9.<br><br>**Rearranging dividends**<br>354 ÷ 6 = 59<br>300 and 54<br>50 and 9<br><br>744 ÷ 4 = 186<br>600 and 100 and 44<br>150 + 25 + 11<br><br>Conjectures examples<br>All prime numbers are odd<br>All odd numbers can be generated from 2 or more prime numbers.<br>- Children will then be able to generalise and provide statements of proof. | Show me<br>Convince me<br>Model of proof<br>Satisfying a rule<br>What else do you know? Use a fact<br>True or false<br>What number is missing?<br>Odd one out<br>Undoing (working backwards)<br>Always, sometimes, never.<br>Variables<br>Logical reasoning<br>Undoing<br>Why?<br>Why not?<br>Always, sometimes, never<br>Make and create general statements.<br>Why? Why not? What if?<br>Conjecture then proves<br>Testing conditions (tests of divisibility/rearranging dividends). |

Kate Burton Mathematics Ltd

# Progression in Reasoning

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<th><u>Models of Proof/Evidence</u></th>
<th><u>Specialising</u><br>is about starting with something general and seeing what it tells us about a specific case.</th>
<th><u>Generalising</u><br>is about starting with specific cases and becoming less specific.</th>
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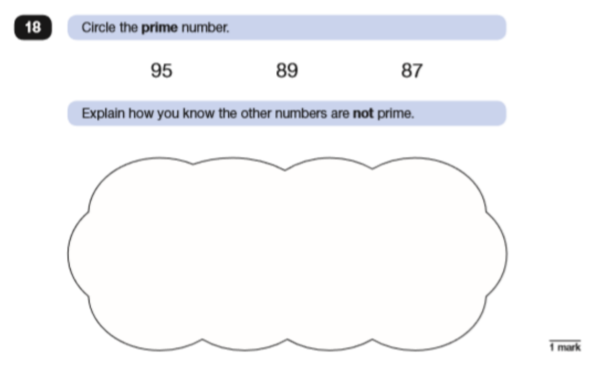

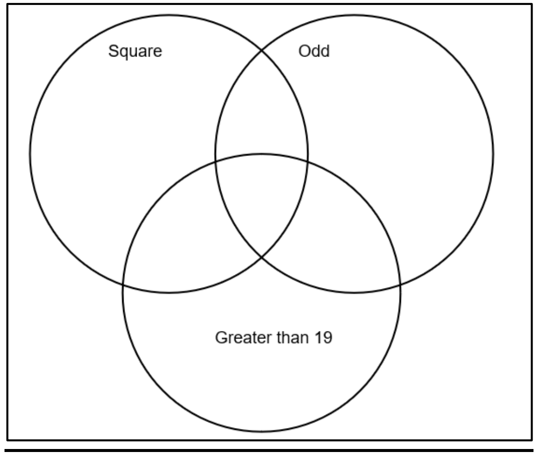

</td>
<td>Any number with a digit sum of a multiple of 3 will divide exactly by 3.<br>87 has a digit sum of 15.<br><br>87 is 3 less than 90, so is a multiple of 3.<br><br><br><br>Any digit sum can be found by adding the digits together.<br>17 has a digit sum of 8.<br>All multiples of 3 have a digit sum of a multiple of 3.<br>18 has a digit of 9, so is a multiple of 3.<br>Digit products are found by multiplying the digits.<br>19 has a digit product of 9.<br><br><u>Square numbers</u><br><br>The square numbers are 1, 4 ,9 16, 25…<br><br><u>True or False</u><br><br>Odd square numbers greater than one have 3 factors.</td>
<td>95 is not prime. Every number that ends on zero or five is a multiple of 5.<br><br>87 has a digit sum of 15. This is a multiple of 3. Any number with a digit sum of a multiple of 3 will divide exactly by 3.<br><br>The product of its digits is 12.<br>The factors of 12 are 1, 12, 2, 6, 3, 4.<br>Numbers with an odd number of factors are square.<br>16, 25, 36.<br><br><u>Square numbers</u><br><br>A square number is the product of number multiplied by itself.<br><br>
<table>
<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>2</td><td>4</td><td>6</td><td>8</td><td>19</td></tr>
<tr><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr>
<tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr>
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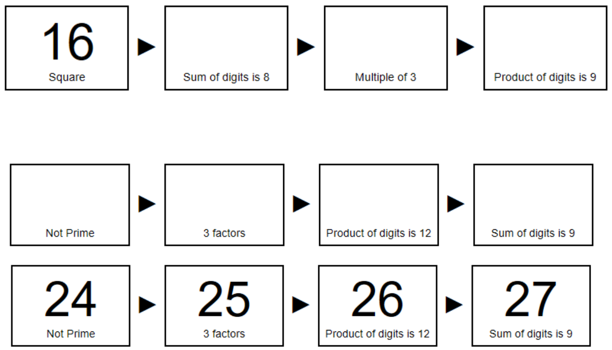
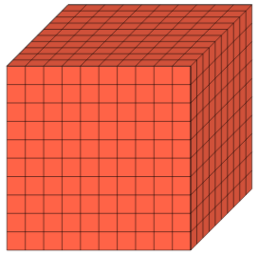
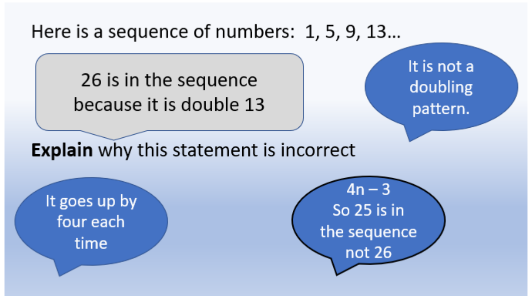
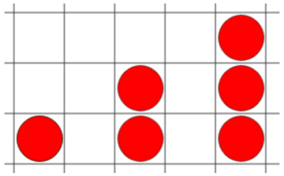
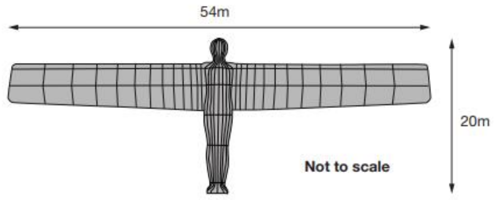
|---|---|---|---|
| | **<u>Consecutive number sequences</u>**<br><br> | **<u>Cube Numbers</u>**<br><br>The cube numbers are 1, 8, 27, 64…<br><br>Erica says 'To find a cube number, you square it first then double your answer. Explain to Erica why she is not correct. | **<u>Cube Numbers</u>**<br>A cube number is the product of a number by itself three times. E.g. 10 x 10 x 10 = 1000<br> |
| **Year 6** | **Key learning** | **Examples of reasoning** | **Vocabulary and thinking terms** |
| Y6 | To apply conceptual knowledge to make generalisations, conjecture relationships and provide sophisticated models of proof, **including formula** and reasoned argument.<br><br><br><br>Expression: 2a + 4b<br>Equation: 3x – 5 = 20<br>Formula: P = 2a + 2b | **We now focus on the third level of proof – formula and expressions for linear sequences**<br><br>**<u>Constructed models of proof</u>**<br>**Concrete**<br><br><br>**Examples to satisfy the rule**<br>1 + 2 + 3= 6<br>2 + 3 + 4= 9<br>3 + 4 + 5= 12<br><br>**Formula**<br>**N + (n+1) + (n+2) = 3n+3**<br><br>***<u>Use a hundred square to track the formula.</u>***<br>In the sequence 1, 4, 7, 10 the 100th term =298. Is this true? Prove it. ***How to solve it;*** | Show me<br>Convince me<br>Model of proof<br>Satisfying a rule<br>What else do you know? Use a fact<br>True or false<br>What number is missing?<br>Odd one out<br>Undoing (working backwards)<br>Always, sometimes, never.<br>Variables<br>Logical reasoning<br>Undoing<br>Why?<br>Why not?<br>Always, sometimes, never<br>Make and create general statements.<br>Why? Why not?<br>Conjecture and proof<br>Testing conditions (tests of divisibility/rearranging dividends). |

**9** The Angel of the North is a large statue in England.

It is 20 metres tall and 54 metres wide.




Ally makes a scale model of the Angel of the North.

Her model is 40 centimetres tall.

How **wide** is her model?

cm

**3** Dev says,


Which expression shows how much money Dev has left?

*a* is the amount of money, in pounds, that Dev gave away.

Tick **one**.

$10 + a$ ☐

$10 \div a$ ☐

$a - 10$ ☐

$10 - a$ ☐

$a \times 10$ ☐

1 mark

100$^{th}$ term = 298

**Expression for linear sequence = 3n - 2**

| | |
|---|---|
| 1 | 1 |
| 2 | 4 |
| 3 | 7 |

Here is a sequence

3, 8, 13, 18, 23

Circle the formula that describes the sequence

4n-1

**5n-2**

3n+5

**Logical reasoning**

Find the generalisation

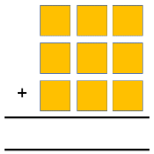
X + 2y = 20

2y = even number so x must equal even

Once you know the value of X, you can find the variables.

**Deductive reasoning**

Using the digits 1 to 9, make three digit numbers that will add as close to 1,500 as you can.



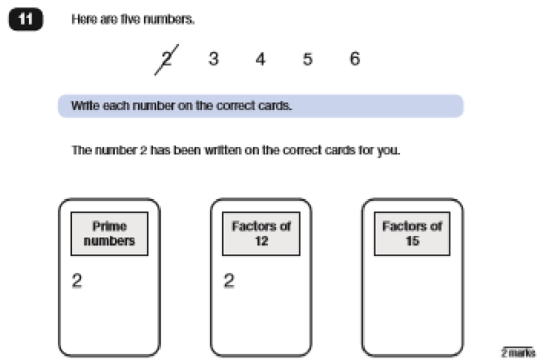
What if…?

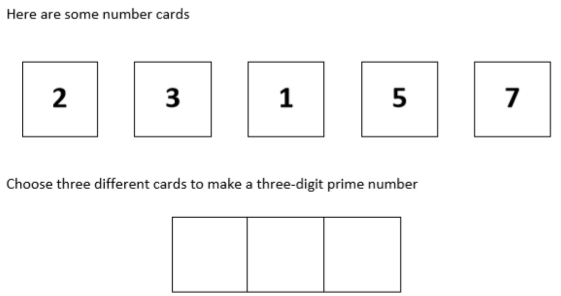
Common differences

Formula for unknowns and variables

# Progression in Reasoning

## Models of Proof/Evidence




## Specialising

is about starting with something general and seeing what it tells us about a specific case.

Factors divide equally into a number.
2,3,4 and 6 are factors of 12.
3 and 5 are factors of 15.
Prime numbers have two factors, one and itself.
2, 3 and 5 are prime numbers.

**Conjecture**
Any prime greater than 3 can be found one before or one after a multiple of 6.
*6n + 1 or 6n -1.* Is this true?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |

**The Test for Prime Numbers**
Is 191 a prime number? Use your tests of divisibility if none of them fit try the next step.

Now divide 191 by 6, what remainder are you left with? Use this table to help you.

| Remainder when we divide by 6 | Can the number be prime? |
|---|---|
| 0 | Must be a multiple of 2 and 3. |
| 1 | May be prime, such as 13. But not always, such as 25. |
| 2 | Must be a multiple of 2. |
| 3 | Must be a multiple of 3. |
| 4 | Must be a multiple of 2. |
| 5 | May be prime, such as 29. But not always, such as 35. |

## Generalising

is about starting with specific cases and becoming less specific.

12 and 15 are in the 3 times tables so cannot be prime as they have more than two factors.

Prime numbers have two factors, one and itself.
2 cannot be a factor of 15 because all multiples of 2 are even.

89 is a prime number. Prime numbers greater than 5 will have a remainder of 1 or 5 when divided by 6.

89 ÷ 6 = 14r5
97 ÷ 6 = 16r1

Prove that 97 is also prime. Tests of divisibility.

Not all numbers with a remainder of 5 or 1 when divided by 6 are prime.

25 ÷ 6 = 4r1 – counter example

# Progression in Reasoning

**23**


Not to scale

The distance from point P to point R is 800 metres.

The distance from point P to point Q is **4 times** the distance from point Q to point R.

Olivia says,


Explain why Olivia is **not** correct.

1 mark

P to R represents the whole.
P to Q represents 4 parts, Q to R represents 1 part, so 5 parts equals the whole P to R.

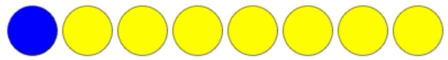
P to Q is four times as long as Q to R.
P to R equals 800m. The ratio is 4:1.

Each part is 800m ÷ 5 = 160m, so P to Q is 640m.

Proportions always relate to the whole.
Ratios always represent the parts.

1 blue bead for every 7 yellow beads on a necklace.
If there are 40 beads;
5:35, 1:7
1 in every 8 beads is blue.

$$\frac{1}{8} \text{ of the beads are blue}$$



**17**

$$x + 2y = 20$$

$x$ and $y$ are whole numbers **less than 10**

What could $x$ and $y$ be?

$x$ =

$y$ =

1 mark

$$x = even \quad 2y = even$$

**Linear equations**

8k = 24

x – 17 = 23

4p – 6 = 2

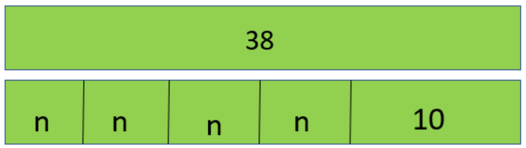
10v + 6 = 46

7 = 4y + 1

Linear equations can have constants and simple variables

$$y = 3x - 6$$

Linear equations can have constants and simple unknowns




4n + 10 = 38